**Final Exam- To Be Done By Yourself**

**MSiA Program, Fall 2023**

**Optimization and Heuristics**

**Due: Wednesday, 12/6/2023, by 11:59 PM CT via Canvas**

**(If you can turn it in earlier—even days earlier, that would be great—and that will prevent last-minute glitches)**

*Instructions*: This Word document is the final exam. Please complete the test in this document and submit it to Canvas when you are finished. Please rename the file to include your name or initials. Also, include your name within the document. The tests are due by Wednesday, but please feel free to turn them in early.

*Scoring*. The final exam is worth 20% of your grade. Questions 1-6 are worth 1 point each and should be short answers. Question 7 is worth 1.5 points, Questions 8, 10, and 11 are worth 2 points each, and Questions 9 and 12 are worth 3 points. Question 13, is an easy half point-- just provide an answer.

***Please complete the test on your own and do not work with anyone else on it. This is not a group assignment.***  However, you may use the text and your notes. If you have any questions, please feel free to contact us.

----------------------------------------------------------------------------

**Name**: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Why is it important for a data scientist or a data science team to know about linear and integer programming? Or why isn't knowing about machine learning, deep learning, and reinforcement learning enough?
2. After this class, how can you more systematically spot optimization problems in the real world?
3. Assume that you have two manufacturing plants that need to make nine different products. Assume that annual demand for each product is 1 million units and that to achieve economies of scale you need to make at least 600,000 units at a single plant. You would also like to have each plant make the same number of units. What will happen to your model if you put in a constraint saying that plant capacity is 4.5 million units and at least 600,000 units of a product must be made at a plant?
4. Why should you be skeptical if a vendor (solving what you know to be solving a linear program with integer variables) presents a solution that claims to optimally solve *any* instance of a problem very fast?
5. In this class, most models were deterministic LPs or IP’s. For this problem, consider the JADE and other network design models or the Beer-Ale problem. In these cases, the underlying LP or IP model did not directly account for uncertainty. However, in each case, the real-world problem we were solving had uncertainty.

A. Why didn’t the underlying models account for the uncertainty?

B. How could these models be used to address the uncertainty without having to change the underlying LP or IP? (hint: don’t make these problems stochastic optimization problems)

1. When solving an integer program, you often have to set an optimization gap (or ‘branch and bound tolerance’ in OpenSolver). What is this gap? (Explain it clearly). And, if you stop a program with a gap of 10%, does that mean you have not found a good solution?
2. This question has two parts. Part A: In the Sports Team scheduling problem, we discussed using column generation. Why can you describe this approach as an “optimization-based heuristic?” Part B: Think about the solution to the revenue management homework problem—the problem with one price and a limited capacity. In class, I showed how to solve it with a simulation. How would you *best* describe my solution approach: was it closest to Optimization-based, Enumeration, Heuristic, or an Optimization-based Heuristic? Provide some explanation.
3. ComplexCo makes a simple product but has some complicated purchasing rules. They purchase a rare earth element they call XT-10. They then process XT-10 and sell it. For our purposes, assume they need to buy 1 unit of raw XT-10 to make 1 unit of demand for the finished good. Assume that they have demand of 170 units for the finished good. They can buy raw XT-10 from one of three sources- A, B, and C. A, B, and C each have a limit of 100 units. Supplier A costs $1 (all $ figures are in millions) per unit, B costs $1.2 per unit, and C costs $1.5. Because of contracts, they have to buy at least twice of much from B as from A. And, they have to buy at least as much from C as from the other two suppliers combined. Formulate this as a linear program and solve it. What is the minimum cost solution? Provide some details on your formulation.
4. Feedco produces two types of cattle feed. They produce it based on the by-products (wheat and corn) rejected by a human food producer. There is a daily limit on the amount of wheat and corn every day—usually between 500 and 1000 pounds. They buy the wheat for $0.50 per pound and the corn for $0.45 per pound. They buy alfalfa as a filler every day for $0.40 with no limit. Both feeds consist totally of wheat, corn, and alfalfa. Feed 1 is their higher-valued product and sells for $1.50 per pound. Feed 2 sells for $1.20. Demand for each type of feed is unlimited. The high-value product must have a lot more wheat and corn than the low-value product, and the wheat is more valued than the corn. But both products have some of all three ingredients. Formulate a Linear Program to maximize Feedco’s daily profit. 1 You’ll need to develop reasonable constraints to make sure Feed 1 is better (has more wheat and corn) than Feed 2. Formulation should clearly show the decision variables, the objective function, and the constraints. The user should be able to adjust the model to create more or less differentiation between Feed 1 and Feed 2. Also, solve the LP (in Python or Excel) with some sample data and provide the optimal solution.
5. In the electricity unit commit model we covered in class, there was a logic constraint that helped ensure that the variables for turning on (“On”), turning off (Off), is on during the period (“Operating Now”), and was on in the last period (“Was Operating”). The constraint for every time period is (On – Off) – (Operating Now – Was Operating) = 0. In a situation where the plant was operating in the last period, and we want it to continue to operate, the Operating Now and Was Operating variables are both 1. To satisfy this constraint, the On and Off variables can be either both be 1 or both 0. Is there a flaw in this constraint? If so or not, please explain.
6. Practice formulating Linear and Integer Programs. Formulate these for optimization:

A. Consider three binary variables: x, y, z. Formulate the two if-then constraints below:

If then

If then

B. Formulate the following if-then constraints where and

If , then

If , then

C. Assume variables A, B, and C are continuous, non-negative decision variables. A sea port can load either 12 A’s per week, or 44 B’s, or 31 C’s. What combinations of A, B and C can be loaded in 8 weeks?

1. You are advising a CEO who has just merged with another company. Let’s call the two legacy companies Division 1 and Division 2. You have a pool of $200 million to invest in the merger. And you have identified 40 potential projects. (You can see the data in the spreadsheet posted with the final). Each project has an expected return to each of the divisions and an expected return to the company as a whole. This latter return is simply the sum of the returns to each division. You note that the two returns are not a 50/50 split—some of the investments will help one of the two divisions much more than the others. The CEO wants to make the right choice of investments to help the company. However, as with all mergers, the politics are tricky. The CEO needs both divisions to feel equal and work together as a team. Therefore, the CEO can’t be perceived as favoring one division over the other. Run an optimization model for the CEO to help make a good decision on how to invest the $200 million. Explain your results.
2. *(Easy Half Point, if you answer it with an explanation):* (A). What are the most important lessons from this class *for you*? What insights or ideas will be best for your future career? (B). What lectures, exercises, or readings were most impactful and memorable from this class?

---------------------------------------------------------------------------------------------------------

The End.

Thanks for your time this Quarter. We hope you learned a lot during course. And, we hope you find good ways to create value with optimization. Good luck with the start of your data science career. Keep in touch.

1 inspired by Wayne Winston’s text book